

Mathematics: Analysis and Approaches – Higher Level

PRACTICE PAPER 6

Compiled by Sotiris Avdalas

Topic 2. Functions

1. [Maximum mark: 6] [without GDC]

The polynomial $p(x)$ is divisible by $(x - 2)$ and has a remainder -6 when divided by $(x + 1)$.

(a) Find the value of a and of b . [4]

(b) Factorise completely $p(x)$ and state its roots. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

2. [Maximum mark: 10] [without GDC]

(a) Solve the equation $|x - 10| = 12 - x$ [5]

(b) Solve the inequality $|x - 8| < x - 6$ [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

3. [Maximum mark: 6] [without GDC]

Draw the graph of $f(x) = |x - 2| - |x| + 2$. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

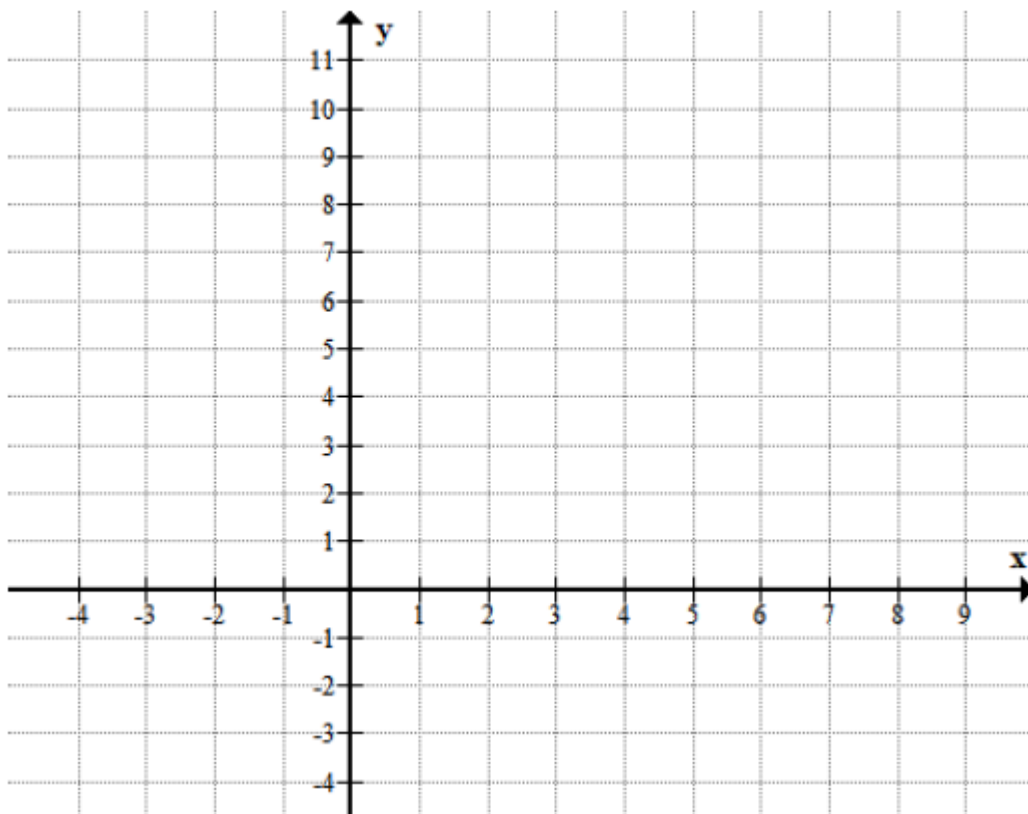
.....

.....

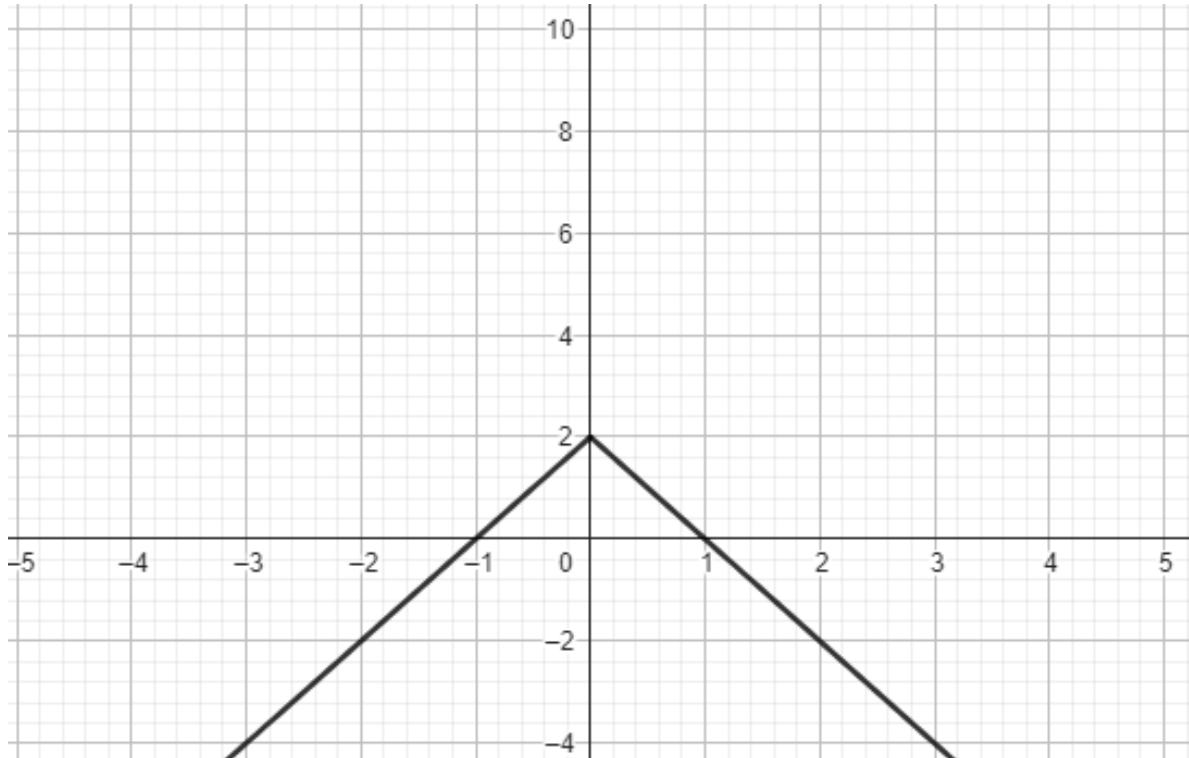
.....

.....

.....



4. [Maximum mark: 5] [without GDC]



The graph of $y = f(x)$ is shown above. On the same diagram, sketch the graph of $y = f(x)^2$ [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

5. [Maximum mark: 12] [without GDC]

Let a and b be the roots of the quadratic $f(x) = x^2 - 7x + 1$.

(a) Write down the values of (i) $a + b$ (ii) ab . [3]

(b) Find the values of (i) $a^2 + b^2$ (ii) $a^3 + b^3$ [3]

(c) Find a quadratic with integer coefficients which has roots $2a+3, 2b+3$ [3]

(d) Find a quadratic with integer coefficients and roots $\frac{1}{a^2}, \frac{1}{b^2}$ [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

6. [Maximum mark: 6] [without GDC]

Solve the inequalities

(a) $(x - 3)^{2025}(2 - 4x)^7(x + 1)^2 > 0$ [3]

(b) $\frac{(x^2 - 9)(2 - x)}{x + 1} \leq 0$ [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

7. [Maximum mark: 7] [without GDC]

Find all the asymptotes (horizontal, vertical, or oblique) of

$$f(x) = \frac{4x^3 + 1}{x^2 - 3x + 2} . \quad [7]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

8. [Maximum mark: 6] [without GDC]

(a) Express $y = 2x^2 - 8x + 1$ in the form $y = 2(x - c)^2 + d$. [3]

The graph of $y = x^2$ is transformed into the graph of $y = 2x^2 - 8x + 1$ by the transformations

a vertical stretch with scale factor k followed by

a horizontal translation of p units followed by

a vertical translation of q units.

(b) Write down the value of

(i) k ;

(ii) p ;

(iii) q . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

