Mathematics: Analysis and Approaches - Standar level

PRACTICE PAPER 5   **Compiled by Sotiris Avdalas**

Topic 2. Functions

1.   [Maximum mark: 6] [without GDC]

Point P has coordinates (-2, 4), and point Q has coordinates (4 , -8) . Point M is the midpoint of [PQ].

(a) Find the coordinates of M.   [2]

Line L is perpendicular to [PQ] and passes through M.

(b) Find the gradient of L.   [2]

(c) Hence, write down the equation of L .   [2]

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2. **[Maximum mark: 7] [without GDC]**

The function f is defined by $f(x) = \dfrac{4x+1}{x-2}$, $x \in R$.

(a) Find the zero of f(x). [2]

(b) For the graph of y = f (x), write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

(c) Find f $^{-1}$(x) , the inverse function of f (x) . [3]

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3. **[Maximum mark: 6] [without GDC]**

Find the points of intersection between $y=x^2-5x+2$ and $y=x-3$

and sketch a graph to demonstrate the result.

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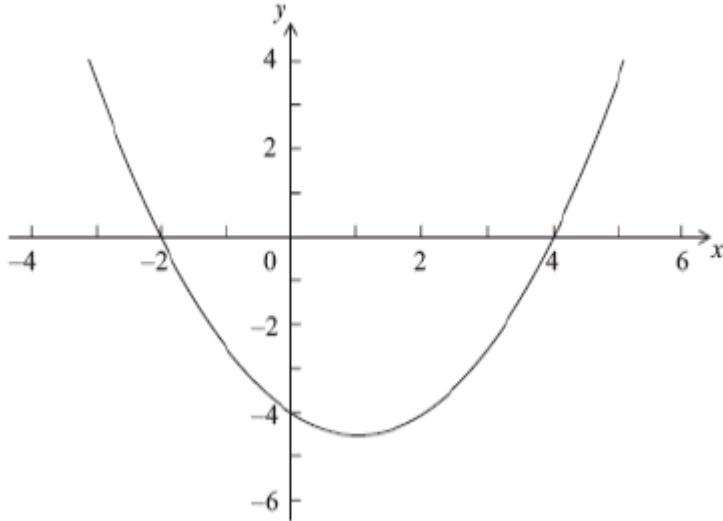...

4. **[Maximum mark: 5] [without GDC]**

Let $f(x) = a(x-x_1)(x-x_2)$

(a) Write down the value of x_1 and of x_2. [2]

(b) Write down the equation of the axis of symmetry. [1]

(c) Find the value of a. [2]

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5. **[Maximum mark: 6] [without GDC]**

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = ax + b$, where $a, b \in \mathbb{R}$ and $g(x) = x^2 - 2x + 3$. Find the two possible functions f such that

$(g \circ f)(x) = 4x^2 - 8x + 6$.

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6. **[Maximum mark: 6] [without GDC]**

Solve the equations

(a) $8^{x+2} = \dfrac{1}{4^{2x}}$ [2]

(b) $\ln^2(x-1)+2\ln(x-1)=3$ [2]

(c) $3^x = 10^{x-2}$ [2]

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7. **[Maximum mark: 8] [without GDC]**

Let $f(x) = \sqrt{5 - x} + 2$

(a) Write down the domain and the range of f(x).   [3]

(b) Find $f^{-1}(x)$.   [3]

(c) The graph of f is translated by the vector $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ to give the graph of h.

The point (1, 2) on the graph of f is translated to the point P on the graph of h. Find the coordinates of P.   [2]

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8. **[Maximum mark: 6] [without GDC]**

(a) Express $y = 3x^2 - 12x + 9$ in the form $y = 3(x - c)^2 + d$. [3]

The graph of $y = x^2$ is transformed into the graph of $y = 3x^2 - 12x + 9$ by the transformations

a vertical stretch with scale factor k followed by

a horizontal translation of p units followed by

a vertical translation of q units.

(b) Write down the value of

(i) k;

(ii) p;

(iii) q. [3]

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9. **[Maximum mark: 7] [without GDC]**

The quadratic equation $x^2 + (k-2)x + (k+1) = 0$ has two equal real roots.

(a) Find the possible values of k. (5)

(b) Write down the values of k for which $x^2 + (k-2)x + (k+1) = 0$ has two real roots. (2)

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10. **[Maximum mark: 10] [with GDC]**

Consider the function $y = f(x)$ with $f(x) = 4e^{0.2x} + 1$

(a) Write down the domain of f. [1]

(b) Find the y-intercept of the graph. [2]

(c) Find

(i) $f(2)$ correct to 3sf.

(ii) $f^{-1}(10)$ correct to 3sf. [3]

(d) Find the first integer value of x for which the value of y will exceed 90. [2]

(e) Write down the range of f [2]

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11. **[Maximum mark: 8] [without GDC]**

The functions $f(x)$, $g(x)$ are defined by $f(x) = e^x$, $g(x) = \ln(2x - 4)$.

Find

(a) $f^{-1}(x)$ and $g^{-1}(x)$. [3]

(b) (fog)(x), (gof)(x), (fof)(x). [3]

(c) $(fog)^{-1}(x)$. [2]

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12. **[Maximum mark: 6] [without GDC]**

The function f(x) has domain $\{x/x \geq 8\}$ and range $\{y/-8 < y \leq 12\}$.
Find the domain and the range of $g(x) = 2f(2x - 3) + 4$.

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