

Mathematics: Analysis and Approaches - Standard level

PRACTICE PAPER 3

Compiled by Sotiris Avdalas

Topic 1: Number and algebra

1. [Maximum mark: 4] [without GDC]

Consider the expansion of $\left(4x^2 - \frac{1}{x}\right)^9$.

(a) How many terms are there in this expansion? [1]

(b) Find the constant term in this expansion. [3]

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2. [Maximum mark: 4] [without GDC]

Evaluate the infinite series $\log\sqrt{2} - \log^4\sqrt{2} + \log^8\sqrt{2} - \log^{16}\sqrt{2} + \dots$

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3. [Maximum mark: 4] [with GDC]

How much should I invest now to yield \$20000 in 4 years' time, if the money can be invested at a fixed rate of 2.2% p.a. compounded quarterly?

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4. [Maximum mark: 8] [without GDC]

The amount, in milligrams, of a medicinal drug in the body t hours after it was injected is given by $f(t) = 10(0.8)^t$, $t \geq 0$. Before this injection, the amount of the drug in the body was zero.

(a) Write down the initial dose of the drug [2]

(b) Find the percentage of the drug that leaves the body each hour. [3]

(b) Calculate the amount of the drug remaining in the body 4 hours after the injection. [3]

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5. [Maximum mark: 6] [with GDC]

A ball is dropped from a height of 2 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 80 % of the previous maximum height.

(a) Find the maximum height reached by the ball after it has bounced for the fifth time. [3]

(b) Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fifth bounce. [3]

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6. [Maximum mark: 6] [with GDC]

On 1 January 2024, Anna deposited \$ 2000 into a bank account with an annual interest rate of 5 % , compounded monthly. At the end of January, and the end of every month after that, she deposits \$ 100 into the same account.

(a) Calculate the amount of money in her account at the start of 2026.

Give your answer to two decimal places. [3]

(b) Find how many complete months, counted from 1 January 2024, it will take for Anna to have more than \$ 6000 in her account. [3]

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7. [Maximum mark: 5] [without GDC]

Find the sum of the integers from 10 to 800 that are not multiples of 4 or 9.

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8. [Maximum mark: 6] [without GDC]

One stage has 23 rows of seats. In the bottom row there are 800 seats and in the top row there are 1 240 seats. The number of seats increases from row to row by the same number of seats. Find how many seats the stadium has in total and how many seats are in the middle row.

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9. [Maximum mark: 4] [with GDC]

When the expression $(2 + ax)^8$ is expanded, the coefficient of the term in x^4 is 90720. Find the value of a .

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10. [Maximum mark: 4] [without GDC]

Find the term x^3 in $(3x - 2)(x + 2)^8$

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11. [Maximum mark: 6] [without GDC]

Consider the geometric sequence U_n with $u_3 = 4$, $u_5 = 16$ and ratio $r > 0$.

(a) Find u_1 and ratio r and the sum of first 10 terms. [2]

(b) Show that $V_n = \frac{1}{u_n}$ is also geometric sequence, find v_1 , ratio r' and the sum of first 10 terms. [3]

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12. [Maximum mark: 6] [with GDC]

If $\frac{a+b}{b} = 4$ and $\frac{c}{d-c} = \frac{1}{4}$, $a, b, c, d \in R$, $b \neq 0, c \neq d$ then evaluate

$$E = \frac{ac+bc}{bd-bc}$$

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