Mathematics: Analysis and Approaches - Higher level

PRACTICE PAPER 4 **Compiled by Sotiris Avdalas**

Topic 1: Number and algebra

1. [Maximum mark: 4] [without GDC]

Find the equation of the quadratic which passes through the points (-1, 8), (2, -1), and (3,0).

2. [Maximum mark: 4] [without GDC]

Solve the following system of equations:

$x - 2y + z = 1$ and $2x + 2y - z = 2$ and $4x - 8y + 4z = 4$

3. [Maximum mark: 6] [without GDC]

Using mathematical induction, prove that $n! > 2^n$, $n \in Z, n \geq 4$.

4. [Maximum mark: 8] [without GDC]

Using mathematical induction, prove that $9^{n+1} - 8n - 9$ is divisible by 64, for n = 1,2,3,…..

5. [Maximum mark: 4] [without GDC]

Prove that if z is a complex number, then both z + z* and zz* are real.

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6. [Maximum mark: 4] [without GDC]

Find the real cubic polynomial with zeros: -2 and 1 + i

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7. [Maximum mark: 10] [without GDC]

Let $z = 2 - 2i$

(a) Find the polar form *r*cisθ of the complex numbers *z* and -*z* . [4]

(b) Find the polar form *r*cisθ of the complex numbers $\overline{z}$ and -$\overline{z}$. [4]

(c) Represent the four complex numbers above on the Complex plane. [2]

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8. [Maximum mark: 9] [without GDC]

Suppose z=1+i and w=1-$\sqrt{3}i$

(a) Write z and w in polar form. [3]

(b) Hence find zw in polar form. [3]

(c) Describe the transformation to z when it is multiplied by w. [3]

9. [Maximum mark: 4] [without GDC]

Find the sum $i + i^2 + i^3 + \cdots + i^{20}$

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10. [Maximum mark: 4] [without GDC]

Solve the equation $z^4 = 16\left(cos\frac{4\pi}{3} + isin\frac{4\pi}{3}\right)$.

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11. [Maximum mark: 6] [without GDC]

Prove that arg(z") =nargz for all complex numbers z and rational n.

12. [Maximum mark: 6] [without GDC]

If b is a multiple of a , prove then that $2^b - 1$ is a multiple of $2^a - 1$, $a, b \in Z$.